Grade 8

Forward With Math

A renewed approach to teaching and learning mathematics

LEARNING SITUATION

It Depends!

AT A GLANCE

In this learning situation, the student will have to create a game of chance for the fair that must include both a minimum of 3 dependent events and a minimum of 3 independent events.

During consolidation, students use their knowledge of probability to create a game and compare the probability of winning for both dependent and independent events.

LIST OF ACRONYMS

PS Problem Solving

C Connecting

RP Reasoning and Proving

TS Selecting Tools and Strategies

CO Communication

REP Representing

REF Reflecting

OVERALL AND SPECIFIC EXPECTATIONS

Number

B1 Demonstrate an understanding of numbers and make connections to the way numbers are used in everyday life.

B1.4 Use fractions, decimal numbers, and percents, including percents of more than 100% or less than 1%, interchangeably and flexibly to solve a variety of problems.

B2 Use knowledge of numbers and operations to solve mathematical problems encountered in everyday life.

B2.5 Add and subtract fractions, using appropriate strategies, in various contexts.

B2.6 Multiply and divide fractions by fractions, as well as by whole numbers and mixed numbers, in various contexts.

Data

D2 Describe the likelihood that events will happen, and use that information to make predictions.

D2.1 Solve various problems that involve probability, using appropriate tools and strategies, including Venn and tree diagrams.

D2.2 Determine and compare the theoretical and experimental probabilities of multiple independent events happening and of multiple dependent events happening.

Notes: Specific expectations in bold type are covered in the learning situation.

Specific expectations not in bold type are only covered in the mini lessons associated with the learning situation.

LEARNING GOALS

At the end of this learning situation, the student will be able to:

- solve various probability problems;
- determine and compare the theoretical and experimental probabilities of an event happening;
- determine and compare probabilities for independent events and for dependent events.

POSSIBLE SUCCESS CRITERIA

During this learning situation, students are invited to develop the success criteria. Here are examples:

- I choose the right data and the appropriate operations.
- I can interpret the results according to the context presented.
- I present my reasoning and organize my calculations, leaving traces.
- I use the conventions and terminology under study.

MATERIALS

- calculator;
- coloured tokens, optional;
- coloured beads, optional;
- erasable felt-tip pens;
- grid paper;
- dice, optional;
- playing cards, optional.

TYPES OF REASONING (LINKED TO SUPPORT DOCUMENTS)

Spatial Reasoning

Targeted spatial skills:

- Creating and reading maps, graphs, and other visual forms of data (determining probabilities using tree diagrams).

Proportional Reasoning

Concepts related to proportional reasoning:

- Equivalence and comparison of fractions (using fractions to establish and compare the probabilities of several events).
- Understanding fraction operations (simplifying and adding fractions to understand the probabilities of several events).

Algebraic Reasoning

Concepts related to algebraic reasoning:

- Understanding the part-whole relationship (fraction) (determining the number of favourable outcomes over the number of possible outcomes in order to determine the probabilities).

Strands	Mini Lessons	Mathematical Concepts
Number	• Expressing values as fractions, decimals and percents	• Relationships between fractions, decimal numbers and percents
	• Adding and subtracting fractions	• Adding and subtracting fractions
	• Multiplying and dividing fractions by fractions, as well as by whole numbers and mixed numbers	• Multiplying fractions
Data	• Solving probability problems using Venn diagrams and tree diagrams*	• Problem solving and comparing probabilities
	• Comparing the theoretical and experimental probabilities of multiple independent events happening and of multiple dependent events happening	

* Mini Lessons marked with an asterisk present the key concepts covered in this learning situation. It is important to ensure that each student has a good understanding of these concepts.

IT DEPENDS!

What do you notice?

BEFORE LEARNING (WARM-UP)

OBSERVING

Action

- Group students into teams. Show them the illustration representing the learning situation, then ask them the following question:
  What do you notice?
- Invite students to record their observations individually. Ask them to discuss them with their team members.
- Lead a discussion with students about the observations noted.

Possible Observations

- The student does not understand the context of the illustration shown.
- The student notes one or two observations only.
- The student makes several observations related to mathematical concepts.

Possible Interventions

- What grabs your attention first? (PS)
- What details catch your attention? (PS)
- What does this problem remind you of in your daily life? (C)

Possible Answers

- I see several stations in a common room.
- I see elements related to games of chance, such as dice, coloured tokens and cards.
- I notice an empty station.
- I notice the notations of probability (tree diagram) in the bubble.
- I understand it's a math fair.
- I understand that the concepts covered are related to probability.

TARGETING A QUESTION

Action

- Ask the teams to formulate one or two questions that the class could answer following their observations.
- Lead a discussion to allow students to discuss the questions formulated.
- Introduce students to the target question (problem to be solved) in the orange box.

Possible Observations

- The team can not formulate a question properly.
- The team's question is too simple.
- The team formulates questions inspired by the illustration and related to mathematical concepts.

Possible Interventions

- What are you trying to demonstrate? (RP)
- Where have you seen this before? (C)
- What tool would be useful to solve this problem? (TS)

Possible Answers

- What needs to be created to fill the empty station?
- Can you compare dependent and independent probabilities?
- What is the probability of drawing a red card among the cards that have not been drawn if we use a complete deck of cards?

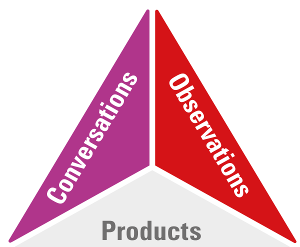
TARGET QUESTION

Can you create a probability game for the fair that requires participants to have basic knowledge in order to maximize their chances of winning? Your game must include a minimum of 3 independent and 3 dependent events.

ESTIMATING OR PREDICTING

Action

- Ask students to sketch one or more models of probability activities, using the manipulatives provided. Tell them that they must determine whether the events are independent or dependent.
- Ask students to identify and write down the missing data essential to solving the problem as they build their model.
- Invite a member of each team to present the probability activities to the class and lead a discussion to highlight the similarities and differences between independent and dependent events.



Triangulation of Assessment

Possible Observations

- The team makes inaccurate predictions.
- The team forgets to include some important elements in their model.
- The team makes an estimation based on realistic and relevant data.
- Due to the lack of data, the team is unable to make a prediction.

Possible Interventions

- What manipulatives can help you find an answer? (TS)
- What do you want to communicate? (CO)
- How can you visually represent the situation or problem to be solved? (REP)

Possible Answers

- I think some activities will have more events than others.
- I believe that probabilities will be more complex when there are more events.
- I assume the probabilities will change for each activity.
- I need to produce a model for the different events in my game.

IDENTIFYING MISSING DATA

Action

- Ask students the following question: What information is needed to solve the problem in the **Target Question**?
- Tell students that there are several ways to solve the problem. Invite them to identify missing data by conducting research or provide them with the following information:
  - Here are some examples of probability activities that could be used by students.

  Independent events.
  - Roll 1 die (6 outcomes)
  - Roll 2 dice (36 outcomes)
  - Roll 3 dice (216 outcomes)
  - Heads or tails
  - Drawing cards with replacement (54 outcomes)
  - Drawing coloured beads with replacement
  - A wheel of fortune

  Dependent events
  - Drawing cards without replacement
  - Drawing coloured beads without replacement
  - Drawing numbered beads without replacement
  - Drawing gumballs without replacement
  - Suggest that students consult the following website, which allows them to create tree diagrams

Possible Observations

- The student was unable to determine the information needed to solve the problem.
- The student has difficulty recognizing useful information when trying to identify missing data.
- The team recognizes the necessary information and most of the missing data.

Possible Interventions

- What software could be useful to answer the question? (TS)
- Can you create a model of the problem? (REP)
- Can you find similar games to help you with your calculations? (C)

Possible Answers

- I need to know all the possible outcomes for the stages of my game. For example, how many faces for a die or how many playing cards.
- I need to know the difference between dependent and independent events.
- I understand that depending on the type of event, the probabilities will change.
- I understand the person playing can improve their chances of winning if they know the basic elements of probability.

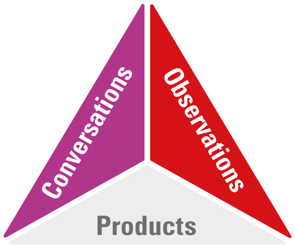
ACTIVE LEARNING (EXPLORATION)

SOLVING

Action

- Allow students the time required to work, think, and determine how to solve the problem by carrying out various experiments.
- Observe teams as they work and identify those who are experiencing difficulties. At the appropriate time, present them with the following mini lesson(s): **Solve Probability Problems Using Venn Diagrams and Tree Diagrams, Compare the Theoretical and Experimental Probabilities of Several Independent and Dependent Events.** Mini lessons will allow students to address, revise, clarify or deepen the concepts necessary to solve the problem.

  Note: Depending on the strategy the students have chosen, it may be necessary to review these mini lessons with them: **Express Values as Fractions, Decimal Numbers and Percents, Adding and Subtracting Fractions, Multiplying Fractions by Fractions, Whole Numbers and Mixed Numbers.**
- Allow these students to continue their work.



Possible Observations

- There is too much data, and the student cannot clearly target the steps necessary to solve the problem.
- The team forgets to include important data.
- The team targets the steps necessary to develop its model by including important data.

Possible Interventions

- Can you create a model of the problem to solve? (PS)
- How can your knowledge of these mathematical concepts help you solve the problem? (C)
- How does your knowledge of numbers allow you to represent the solution in different forms (for example, decimal numbers, fractions, percents)? (REP)

Possible Answers

- Many answers are possible depending on the data used.
- Computational strategies may vary.

COMPARING, EXCHANGING, AND IMPROVING

Action

- Ask the teams to compare their results with those of another team.
- Post the games of chance on a classroom wall or display them and offer students the opportunity to comment on them and ask questions to the different teams by placing sticky notes on the solutions that interest them. Ensure that questions and comments are constructive and related to the pedagogical intent of the learning situation.
- Get the students thinking by asking them the following questions: Compare your results with those of another group. Are you satisfied with your solution? If yes, explain why.
  Otherwise, modify your solution.

Possible Observations

- The student is unable to compare results and pinpoint errors.
- The student does not understand their mistakes or does not think to understand them.
- The student cannot correct their work.
- The team can compare its results with others and identify missing elements.

Possible Interventions

- Can other students understand your reasoning? (RP)
- Do you use the right symbols and mathematical conventions to express your solution? (CO)
- How can you explain your approach (for example, using words, diagrams, gestures, symbols)? (CO)

Possible Answers

- I don't understand my mistake.
- I got the same answer as another team, but I don't understand their strategy.
- Using the comment of another student, I improved the communication of my reasoning, or I corrected an error in my probability calculations related to my game.

CONSOLIDATION OF LEARNING

PRESENTING SOLUTIONS

Action

- In order to lead a mathematical exchange, choose 2 solutions containing specific elements related to the pedagogical intent. Ask the teams involved to present their solution and reasoning to the class.
- Focus on the important elements of the approaches presented by the teams in order to help students progress in their learning. To guide the discussion, it is possible to frame the targeted elements using masking tape or a paper frame.
- If necessary, suggest another possible solution to the class, making sure to make connections with the students' approaches.

Possible Observations

- The solution proposed by a team is false and confusing.
- The proposed solution is not well organized. Students did not use correct mathematical conventions.
- The solution presented by the team is generally good but has missing elements.
- The solution presented is complete and contains most of the important data.

Possible Interventions

- Were the strategies used effective? (PS)
- What was the best strategy to solve the problem? (TS)
- Which stages of the work were the most challenging? Why? (PS)

POSSIBLE SOLUTION

Here is a possible example of a game and the calculations for the theoretical probabilities.

Title: It's Probably the Right Sum

In this game, the participant must draw a card at random. On the card there is a sum between 3 and 21 inclusively. The participant must determine which game offers the best probability of obtaining the amount on their card. If the participant can determine the exact probability of the two situations before making their choice, they are awarded points equivalent to the value of the probability of the event as a percentage. In addition, if the participant obtains the desired sum, after having performed the experiment, he or she can multiply his or her score by the amount obtained.

Here are the two games:

Game 1 – Three Dice (Independent Events)

In this game, the participant rolls three dice and adds up the three sides that come up. For example, in the following image, the sum would be 9.

Game 2 – Eight Cards (Dependent Events)

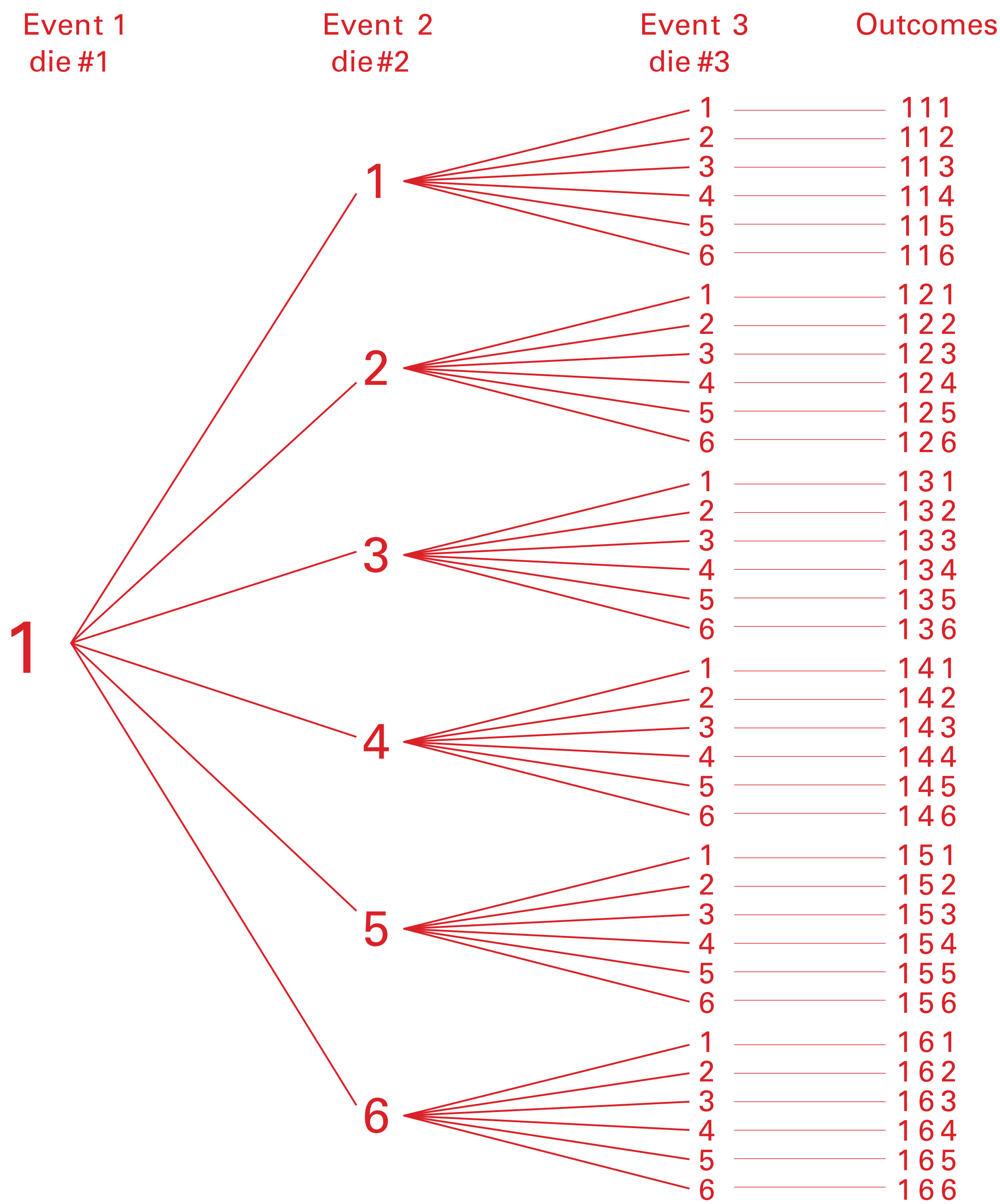
The student must draw three cards from a deck of eight cards. In the deck, there are cards from 1 (Ace) to 8. The student cannot put the cards back in the deck after the first and second draws.

Calculations for theoretical probabilities

Game 1 – Three Dice

I know that I can create a tree diagram to be able to find all outcomes. Since there are three dice, I know there will be three events in my tree diagram. For each outcome of Event 1, I must consider each outcome of Event 2 and for each outcome of Event 2, I must consider each outcome of Event 3.

Here is an example of all the possible outcomes if I get a 1 when rolling the first die.



I see that if I roll a 1 in Event 1, there are 36 different outcomes. I can repeat the process if I roll a 2, 3, 4, 5, and 6 on the first die. Therefore I'm going to get 216 different outcomes. For each of these outcomes, I want to find the sum of the three dice since that is what interests me for the game. For the example above, I would obtain the following sums:

Results of the 3 Dice	Associated Sums	Probabilities for Each Sum in This Situation (Getting 1 With the First Die)		
		Sum	Fraction	Percents
1, 1, 1	3	3	$\frac{1}{36}$	2.78 %
1, 1, 2	4	4	$\frac{2}{36}$	5.56 %
1, 1, 3	5	5	$\frac{3}{36}$	8.33 %
1, 1, 4	6	6	$\frac{4}{36}$	11.11 %
1, 1, 5	7	7	$\frac{5}{36}$	13.89 %
1, 1, 6	8	8	$\frac{6}{36}$	16.67 %
1, 2, 1	4			
1, 2, 2	5			
1, 2, 3	6			
1, 2, 4	7			
1, 2, 5	8			
1, 2, 6	9			
1, 3, 1	5			
1, 3, 2	6			
1, 3, 3	7			
1, 3, 4	8			
1, 3, 5	9			
1, 3, 6	10			
1, 4, 1	6			
1, 4, 2	7			
1, 4, 3	8			

Results of the 3 Dice	Associated Sums	Probabilities for Each Sum in This Situation (Getting 1 With the First Die)		
		Sum	Fraction	Percents
1, 4, 4	9			
1, 4, 5	10			
1, 4, 6	11			
1, 5, 1	7			
1, 5, 2	8			
1, 5, 3	9			
1, 5, 4	10			
1, 5, 5	11			
1, 5, 6	12			
1, 6, 1	8			
1, 6, 2	9			
1, 6, 3	10			
1, 6, 4	11			
1, 6, 5	12			
1, 6, 6	13			

I am aware that these 36 outcomes are not all the possible outcomes. To get the full theoretical probabilities, I need to do this for all possible outcomes, namely all 216 outcomes.

Here are the results for the 216 sums

Three Dice		
Sums	**Probabilities Expressed in Fractions**	**Probabilities Expressed in Percents**
3	$\frac{1}{216}$	0.5 %
4	$\frac{3}{216}$	1.4 %
5	$\frac{6}{216}$	2.8 %
6	$\frac{10}{216}$	4.6 %
7	$\frac{15}{216}$	6.9 %
8	$\frac{21}{216}$	9.7 %
9	$\frac{25}{216}$	11.6 %
10	$\frac{27}{216}$	12.5 %
11	$\frac{27}{216}$	12.5 %
12	$\frac{25}{216}$	11.6 %
13	$\frac{21}{216}$	9.7 %
14	$\frac{15}{216}$	6.9 %
15	$\frac{10}{216}$	4.6 %
16	$\frac{6}{216}$	2.8 %
17	$\frac{3}{216}$	1.4 %
18	$\frac{1}{216}$	0.5 %

Game 2 – Eight Cards (three cards without replacement)

I know I can create a tree diagram to find the outcomes. Since I'm drawing from the eight-card pack three times, I know that there will be three events in my tree diagram. For each outcome of Event 1, I have to consider each outcome of Event 2, and for each outcome of Event 2, I have to consider each outcome of Event 3. However, since I don't put the cards back after Draw 1 and Draw 2, I have to make sure I create a tree diagram accordingly. This means that on the first draw, there are eight possible outcomes. On the second draw, there are only seven possible outcomes, since the first card drawn is not replaced. Finally, on the third draw, only six possible outcomes remain, since the two cards already picked are no longer in the pack.

Here is an example of a branch of the tree diagram. This branch demonstrates the outcomes if card 3 is drawn during the first draw.

Draw 1	Draw 2	Draw 3	Sum
3	A	2	6
		4	8
		5	9
		6	10
		7	11
		8	12
3	2	A	6
		4	9
		5	10
		6	11
		7	12
		8	13
3	4	A	8
		2	9
		5	12
		6	13
		7	14
		8	15
3	5	A	9
		2	10
		4	12
		6	14
		7	15
		8	16
3	6	A	10
		2	11
		4	13
		5	14
		7	16
		8	17
3	7	A	11
		2	12
		4	14
		5	15
		6	16
		8	18
3	8	A	12
		2	13
		4	15
		5	16
		6	17
		7	18

I see that if I draw a 3 during Event 1, there are 42 different outcomes. I can repeat the process if I draw cards Ace, 2, 4, 5, 6, 7, 8 to get an overview of the outcomes. Therefore I'm going to get 336 different outcomes. For each of these outcomes, I want to find the sum of the three cards since that is what interests me for the game. For example, for the example above, I would obtain the following sums:

Results of the 3 Cards	Associated Sums	Probabilities for Each Sum for This Situation (Drawing 3 at Event 1)		
		Number of Outcomes per Sum	Fraction	Percents
3, 1, 2	6	6	$\frac{2}{42}$	4.8 %
3, 1, 4	8	7	$\frac{0}{42}$	0 %
3, 1, 5	9	8	$\frac{2}{42}$	4.8 %
3, 1, 6	10	9	$\frac{4}{42}$	9.5 %
3, 1, 7	11	10	$\frac{4}{42}$	9.5 %
3, 1, 8	12	11	$\frac{4}{42}$	9.5 %
3, 2, 1	6	12	$\frac{6}{42}$	14.3 %
3, 2, 4	9	13	$\frac{4}{42}$	9.5 %
3, 2, 5	10	14	$\frac{4}{42}$	9.5 %
3, 2, 6	11	15	$\frac{4}{42}$	9.5 %
3, 2, 7	12	16	$\frac{4}{42}$	9.5 %
3, 2, 8	13	17	$\frac{2}{42}$	4.8 %
3, 4, 1	8	18	$\frac{2}{42}$	4.8 %
3, 4, 2	9			
3, 4, 5	12			
3, 4, 6	13			
3, 4, 7	14			

Results of the 3 Cards	Associated Sums	Probabilities for Each Sum for This Situation (Drawing 3 at Event 1)		
		Number of Outcomes per Sum	Fraction	Percents
3, 4, 8	15			
3, 5, 1	9			
3, 5, 2	10			
3, 5, 4	12			
3, 5, 6	14			
3, 5, 7	15			
3, 5, 8	16			
3, 6, 1	10			
3, 6, 2	11			
3, 6, 4	13			
3, 6, 5	14			
3, 6, 7	16			
3, 6, 8	17			
3, 7, 1	11			
3, 7, 2	12			
3, 7, 4	14			
3, 7, 5	15			
3, 7, 6	16			
3, 7, 8	18			
3, 8, 1	12			
3, 8, 2	13			
3, 8, 4	15			
3, 8, 5	16			
3, 8, 6	17			
3, 8, 7	18			

I am aware that these 42 outcomes are not all the possible outcomes. To get the full theoretical probabilities, I need to do this for all possible outcomes, namely all 336 outcomes.

Here are the probabilities for all outcomes:

Eight Cards		
Sums	**Probabilities Expressed in Fractions**	**Probabilities Expressed in Percents**
3	$\frac{0}{336}$	0 %
4	$\frac{0}{336}$	0 %
5	$\frac{0}{336}$	0 %
6	$\frac{6}{336}$	1.79 %
7	$\frac{6}{336}$	1.79 %
8	$\frac{12}{336}$	3.57 %
9	$\frac{18}{336}$	5.36 %
10	$\frac{24}{336}$	7.14 %
11	$\frac{30}{336}$	8.93 %
12	$\frac{36}{336}$	10.7 %
13	$\frac{36}{336}$	10.7 %
14	$\frac{36}{336}$	10.7 %
15	$\frac{36}{336}$	10.7 %
16	$\frac{30}{336}$	8.93 %
17	$\frac{24}{336}$	7.14 %
18	$\frac{18}{336}$	5.36 %
19	$\frac{12}{336}$	3.57 %
20	$\frac{6}{336}$	1.79 %
21	$\frac{6}{336}$	1.79 %

Now that I know the probabilities for all the outcomes for the Three Dice game as well as for the Eight Cards game, I will be able to choose the game that gives me the best chance of winning. By looking at the possible outcomes for both games, I can determine that the Three Dice game gives me the best chance of winning for a sum of 12 or less while the Eight Cards game gives me the best probability of winning for a sum of 13 or more.

Here is a summary table of the probabilities for the two games.

Sums	Eight Cards		Three Dice	
3	$\frac{0}{336}$	0 %	$\frac{1}{216}$	0.5 %
4	$\frac{0}{336}$	0 %	$\frac{3}{216}$	1.4 %
5	$\frac{0}{336}$	0 %	$\frac{6}{216}$	2.8 %
6	$\frac{6}{336}$	1.79 %	$\frac{10}{216}$	4.6 %
7	$\frac{6}{336}$	1.79 %	$\frac{15}{216}$	6.9 %
8	$\frac{12}{336}$	3.57 %	$\frac{21}{216}$	9.7 %
9	$\frac{18}{336}$	5.36 %	$\frac{25}{216}$	11.6 %
10	$\frac{24}{336}$	7.14 %	$\frac{27}{216}$	12.5 %
11	$\frac{30}{336}$	8.93 %	$\frac{27}{216}$	12.5 %
12	$\frac{36}{336}$	10.7 %	$\frac{25}{216}$	11.6 %
13	$\frac{36}{336}$	10.7 %	$\frac{21}{216}$	9.7 %
14	$\frac{36}{336}$	10.7 %	$\frac{15}{216}$	6.9 %
15	$\frac{36}{336}$	10.7 %	$\frac{10}{216}$	4.6 %
16	$\frac{30}{336}$	8.93 %	$\frac{6}{216}$	2.8 %
17	$\frac{24}{336}$	7.14 %	$\frac{3}{216}$	1.4 %

Sums	Eight Cards		Three Dice	
18	$\frac{18}{336}$	5.36 %	$\frac{1}{216}$	0.5 %
19	$\frac{12}{336}$	3.57 %	$\frac{0}{216}$	0 %
20	$\frac{6}{336}$	1.79 %	$\frac{0}{216}$	0 %
21	$\frac{6}{336}$	1.79 %	$\frac{0}{216}$	0 %
Total	$\frac{336}{336}$	100 %	$\frac{216}{216}$	100 %

Here's an example of how to calculate points.

Points before Saïd's draw

Marc 21 points

Saïd 32 points

Saïd draws the card with 13 as the sum. He does the calculations and determines that with the cards, he has a 10.7% chance of success and that with the dice, he has a 9.7% chance of success.

He therefore decides to choose the card game. Since his calculations are correct, he can add 10.7 to his score. He now has $32 + 10.7 = 42.7$ points.

Moreover, by chance, he draws three cards which give a sum of 13. He can therefore multiply his score by 13. He now has $42.7 \times 13 = 555.10$.

CONSOLIDATION OF LEARNING

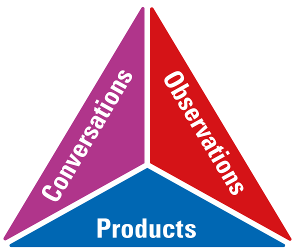
Action

- Lead a discussion with students to determine important learnings by asking them the following questions: Was your estimation relatively correct? What mistakes did you make, or challenges did you face when solving the problem? What have you learned from these mistakes or challenges?
- Give students the opportunity to note important elements related to the types of reasoning and mathematical concepts targeted in this learning situation.
- Develop with the students the success criteria related to the following learning goals:
  At the end of this learning situation, the student will be able to solve various probability problems, determine and compare theoretical and experimental probabilities and determine and compare the probabilities for independent events and for dependent events.
- Ask students to solve the following problem:

A television game offers you a choice in order to win the mystery prize. In a bag, they will place 5 marbles of 2 different colours. You will have to draw 3 times. Invent a game that will give you the best chance of winning without the game appearing to be rigged in your favour.

Note: As you work through this problem, it may be necessary to review some concepts with students through the following mini lessons:
Solving Probability Problems Using Venn Diagrams and Tree Diagrams and Comparing the Theoretical and Experimental Probabilities of Several Independent and Dependent Events.

Note: Gather evidence of student learning, analyze and interpret it to identify strengths and target next steps to help students improve.



Possible Observations

- The student is able to distinguish between dependent and independent events.
- The student can not determine the possible outcomes for a dependent event or for an independent event.
- The student can determine the probabilities and make an appropriate choice.

Possible Interventions

- What would be the best way to represent the solution? (PS)
- Have you ever used a similar strategy? Was it effective? (C)
- How does your solution compare to that of other students? (RP)

POSSIBLE ANSWER

I know that I want the game to be in my favour without it seeming to be too one-sided. Therefore I have to take into account the number of marbles of each colour that will be in the bag. If I place too many marbles of the same colour, the game will appear rigged and will be unacceptable. Therefore I want to have relatively similar quantities, but that are not equal. Therefore I'm going to include three white marbles and two black marbles. The game will therefore focus on drawing white marbles in order to win.

I now want to establish whether it is better to invent a game that has independent events or dependent events. In the case of a game with independent events, the three white marbles will always be in the bag on each of the three draws, but the number of possible outcomes will be greater. In the case of a game with dependent events, the white (or black) marbles will not be put back in the bag, so the number of possible outcomes will be fewer.

In order to make a good decision, I will determine the probabilities for all outcomes for both situations.

Situation 1 – Independent events occur when the marbles are replaced each time. I can quickly determine the number of outcomes by multiplying the outcomes of each draw together.

5 possibilities x 5 possibilities x 5 possibilities = 125 possibilities

Since the game will be based on the outcomes related to the white marbles, I will determine the probabilities for these events.

$$\text{Theoretical Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

First, I will determine the probabilities of drawing 3 white marbles.

For each draw, there are 3 white marbles, for a total of 5 marbles. I therefore have $\frac{3}{5}$.

$$\text{Theoretical probability} = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$$
$$= \frac{27}{125}$$

I will now determine the probabilities of having exactly 2 white marbles.
I must therefore take into consideration the following outcomes:

WWB

WBW

BWW

I'm going to find the probabilities for each event and then add them together afterwards.

$$\text{WWB Probability} = \frac{3}{5} \times \frac{3}{5} \times \frac{2}{5}$$
$$= \frac{18}{125}$$

$$\text{WBW Probability} = \frac{3}{5} \times \frac{2}{5} \times \frac{3}{5}$$
$$= \frac{18}{125}$$

$$\text{BWW Probability} = \frac{2}{5} \times \frac{3}{5} \times \frac{3}{5}$$
$$= \frac{18}{125}$$

The probabilities of drawing exactly two white marbles are $\frac{54}{125}$.

If I combine the results for the 3 consecutive white marbles and for exactly two white marbles, I get $\frac{81}{125}$. This could be an interesting game, since the probability of winning is greater than the probability of losing.

Situation 2 – Dependent events occur when the marbles are not replaced every time. I can quickly determine the number of outcomes by multiplying the outcomes of each draw together. In this case, I have to make sure to remove one outcome from each draw.

All the possibilities = 5 possibilities x 4 possibilities x 3 possibilities
All the possibilities = 60 possibilities

Since the game will be based on the outcomes related to the white marbles, I will determine the probabilities for these events.

$$\text{Theoretical Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

First, I'll determine the probabilities of drawing three white marbles.

For the first draw, there are three white marbles and there are five marbles in total, so I have $\frac{3}{5}$ chance to draw a white marble. For the second draw, there are only two white marbles and four marbles left in total, so I have $\frac{2}{4}$ chances of drawing a white marble. During the third draw, there is one white marble and three marbles in total, so I have $\frac{1}{3}$ chance to draw a white marble.

$$\begin{aligned}\text{Theoretical probability } &= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \\ &= \frac{6}{60}\end{aligned}$$

I am now going to determine the probabilities of having exactly two white marbles. I must therefore consider the following outcomes:

WWB

WBW

BWW

I'm going to find the probabilities for each event and then add them together afterwards.

$$\begin{aligned}\text{WWB Probability } &= \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \\ &= \frac{12}{60}\end{aligned}$$

$$\begin{aligned}\text{WBW Probability } &= \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \\ &= \frac{12}{60}\end{aligned}$$

$$\begin{aligned}\text{BWW Probability } &= \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} \\ &= \frac{12}{60}\end{aligned}$$

The probabilities of drawing exactly two white marbles are $\frac{36}{60}$.

If I combine the results for the three consecutive white marbles and for exactly two white marbles, I get $\frac{42}{60}$. This could be interesting as a game since the chances of winning are greater than the chances of losing.

All I have to do is determine which game gives me the best chance of drawing at least two white marbles. In the case of independent events, the probabilities are $\frac{81}{125}$ to draw two or more white marbles. In the case of dependent events, the probabilities are $\frac{42}{60}$ to draw at least two white marbles. I'm going to convert both fractions to percents so I can determine the probabilities that are most favourable. In the case of the first game, $\frac{81}{125}$ is equivalent to 64.8%. In the case of the second game, $\frac{42}{60}$ is equivalent to 70%.
Thanks to my calculations and my knowledge of probability, I'm going to propose the second game. I'll have a 70% chance of winning the mystery prize.

POSSIBLE EXTENSIONS

1. Determine the probabilities of drawing 3 specific cards from a deck of cards with and without replacement.
2. Invent a probability game with cards or dice.